Text Book of Physics

BY

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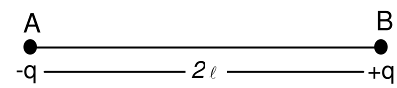
Class:10+2
Unit: I
Topic: Electrostatics

SYLLABUS: UNIT-I

Charges and their conservation. Coulomb's law-force between two charges, forces between multiple charges; superposition principle and continuous charge distribution.
Electric field, electric field due to a point charge, electric field lines; electric dipole, electric field to a dipole; torque on a dipole in a uniform electric field.
Electric flux, statement of Gauss's theorem and its applications to find field due to infinitely long straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell (field inside and outside).
Electric potential, potential difference, electric potential due to a point charged, a dipole and system of charges; equipotential surfaces, electrical potential energy of a system of two point charges and of electric dipoles in an electrostatic field.
Conductors and insulators, free charges and bound charges inside a conductor, Dielectrics and electric polarization, capacitors and capacitance, combination of capacitors in series and in parallel, capacitance of a parallel plate capacitor with and without dielectric medium between the plates, energy stored in a capacitor, Van de Graff generator.

Q15. **(a) What is an electric dipole?**
(b) Define electric dipole moment. Give its direction and SI units.
(c) What does $q_1 + q_2 = 0$ signifies?
(d) What is an ideal dipole?

Ans.a) **Electric dipole**:-



is the system of two equal and opposite charges separated by some distance.
The distance between the charges is called length of dipole.

b) **Electric dipole moment** $(\vec{p})$ →

is the product of magnitude of either charge and distance between them is $\vec{p} = q \times 2\ell$ units in c.m. It is a vector quantity its direction is from – ve to +ve charge. (conventional)

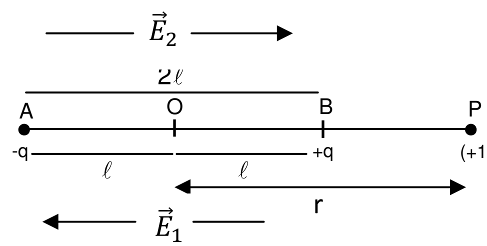
c) $q_1 = -q_2$ i.e dipole

d) If charges are high and distance between them is as small as possible it has cylindrical symmetry.

Q16. Derive an expression for electric field intensity at any point on axial line of dipole.

Ans. <u>**Electric field intensity at any point on axial line of dipole axial line**</u>:-

Line which pass through axis of dipole consider an electric dipole of length '2ℓ' st. 'O' is midpoint of dipole consider a +ve test charge is placed at pt. 'P' at distance 'r' from centre of dipole s.t. op =r



Let $\vec{E}_1$ is electric field intensity at 'p' due to – q charge at 'A'

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{AP^2} \text{ i.e. } \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+\ell)^2} \text{ along}$$

PA. Similarly $\vec{E}_2$ is electric field intensity at 'P' due to +q charge at 'B' $\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(BP)^2}$

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-\ell)^2} \quad \text{along AP}$$

Net Intensity $\vec{E} = \vec{E}_2 - \vec{E}_1 \quad (\because E_2 > E_1)$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{(r-\ell)^2} - \frac{1}{(r+\ell)^2}\right) \text{ along AP}$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{se^2 + \ell^2 + 2\ell r - r^2 - \ell^2 + 2\ell r}{(r^2-\ell^2)^2}\right)$$

$$= \frac{q \times 4\ell r}{4\pi\varepsilon_0 (r^2 - \ell^2)^2} \text{ along AP}$$

$$\vec{E}_{axial} = \frac{q \times 2\ell \times 2r}{4\pi\varepsilon_0 (r^2-\ell^2)^2} \text{ i.e } \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}r}{(r^2-\ell^2)^2}$$

------------ (1)

along AP

$(\because p = q \times 2\ell)$

If dipole is of short length $r >> \ell$

s.t. $r^2 - \ell^2 \cong r^2$ from (1)

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\overrightarrow{pr}}{r^4} \text{ along AP}$$

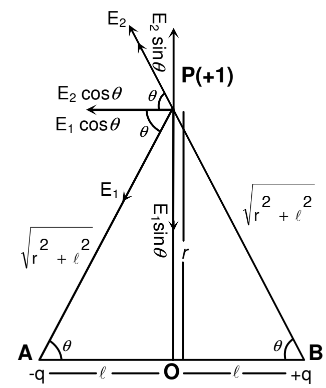
$$\vec{E}_{axial} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3}$$ along – ve to +ve charge i.e in the direction of dipole moment

Q17. Derive an expression for electric field intensity at any point on (a) Equatorial line of dipole

Ans. (a) <u>Electric field intensity at any point on equatorial line of dipole</u> → <u>Equatorial line →</u>

$E_2 \sin\theta$ is the right angle bisector of length of dipole. Let 'p' be any point on equatorial line of dipole AB at distance 'r' from centre of dipole s.t. OP = r

Let $\vec{E}_1$ is electric field intensity at pt 'p' due to '-q' at 'A'.



$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(AP)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+\ell^2)} \quad \text{along PA} \quad \text{------------- } \mathbf{(1)}$$

$\vec{E}_2$ is electric field intensity at 'p' due to 'q' at 'B'

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+\ell^2)} \text{ along BP} \quad \text{-------------- } \mathbf{(2)}$$

from (1) and (2) $|\vec{E}_1| = |\vec{E}_2| = |\vec{E}_1| = |\vec{E}|$

Let $\lfloor \underline{PAB}$ = draw P x || BA st $\lfloor \underline{AP} \times \lfloor \underline{AP} \times \lfloor \underline{AP} \times \lfloor \underline{\times PM = \theta}$ = ?

Resolve $\vec{E}_1 \& \vec{E}_2$ into rectangular compts $E_1 \cos\theta$ and $E_2 \cos\theta$ along 'Px' $E_1 \sin\theta$ along PO & $E_2 \sin\theta$ along Py. $E_1 \sin\theta$ and $E_2 \sin\theta$ are equal and opposite hense cancel each other and compts $E_1 \cos\theta$ and $E_2 \cos\theta$ in same direction so they will add Hense net intensity at

$$'p' = \vec{E}_{eq} = 2E_1 \cos\theta \text{ along Px i.e. } \vec{E}_{eq} = 2 \times \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+\ell^2)} \times \frac{\ell}{\sqrt{r^2+\ell^2}}$$

$$\left(\because \cos\theta = \frac{\ell}{\sqrt{r^2+\ell^2}} \quad \text{IN}\Delta\text{AOP}\right)$$

$$\vec{E}_{eq} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{(r^2+\ell^2)^{3/2}} \quad (\because q \times 2\ell = p) \text{ from +ve to – ve charge}$$

For short dipole $r^2 + \ell^2 \cong r^2$

$$\vec{E}_{eq} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3} \text{ opposite to direction of dipole moment}$$

Compare $\vec{E}_{eq}$ and $\vec{E}_{axil}$ we get $\vec{E}_{axil} = 2\vec{E}_{eq}$

Q18. a) Derive an expression for electric field intensity at any point on axis of uniformly charged ring.
b) When does charged circular ring behave as point charge?

Ans.a) **<u>Electric field intensity at any point on axis of a uniformly charged circular ring</u>** –

Consider a circular ring of radius 'r' on the surface of which charge 'q' is distributed uniformly. Let $\vec{E}_1$ be electric field intensity at pt. 'p' due to small charge 'dq' of small port of ring 'AB'

$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{dq}{a^2}$ along PM where $\left(a = \sqrt{r^2 + x^2}\right)$

Resolve $\vec{E}_1$ into two rectangular compts $E_1 \cos\theta$ along PX and $E_1 \sin\theta$ along '–PY'.

Now by symmetry of ring there will be electric field intensity $\vec{E}_2$ having opt. direction of $\vec{E}_1$.

From fig. it is clear that all vertical compt will cancel each other being equal and oppt. and horizontal compts will add up.

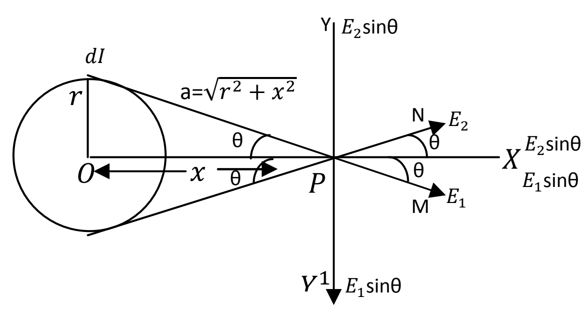
So Net intensity at 'P' $\vec{E} = \sum E_1 \cos\theta$ along PX

In ΔAOP $\cos\theta = \frac{x}{\sqrt{r^2 + x^2}}$

Hence $\vec{E} = \sum \frac{1}{4\pi\varepsilon_0}\frac{dq}{\left(r^2 + x^2\right)} \times \frac{x}{\sqrt{r^2 + x^2}}$

$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{qx}{\left(r^2 + x^2\right)^{3/2}}$ along PX

------------- (1)



<u>Special case</u>:-

(i) Point 'P' lies at centre of ring x =0

So $\vec{E} = 0$

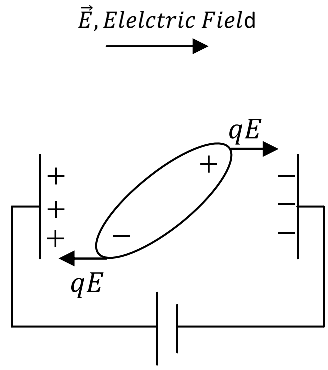
(ii) Point 'p' lines very for away s.t. x >>r

$r^2 + x^2 \cong x^2$

So $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{qx}{x^3}$ i.e. $\frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$

b) When pt P is very -2 far away.

Q19. Derive an expression for torque acting on an electric dipole in a uniform two dimensional electric field.



Ans. **<u>Torque</u>** :- τ = r × F .2

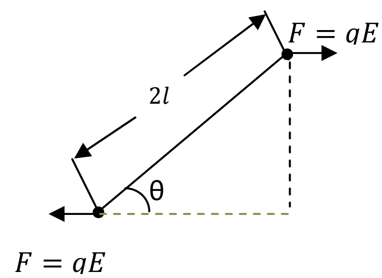
= r F sin θ .2

= (2*l*) (qE) sin θ

= PE sin θ

In the vector form, we can rewrite this eqn. as

$$\vec{\tau} = \vec{P} \times \vec{E}$$



<u>Special cases</u>:-

<u>Case I</u>:- **θ = 0º**

τ = p E sin 0º = 0

The dipole is in *stable equilibrium.*

<u>Case II</u>:- **θ = 180º**

τ = p E sin 180º = 0

As such, the dipole will be in an *unstable equilibrium*.

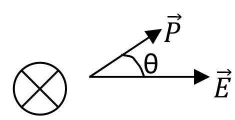
<u>Case III</u>:- **θ = 90º**

τ = p E sin 90º

= p E

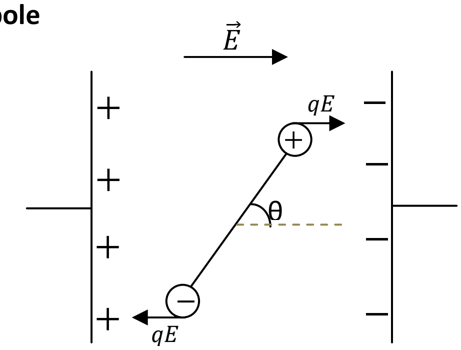
The torque will be maximum,

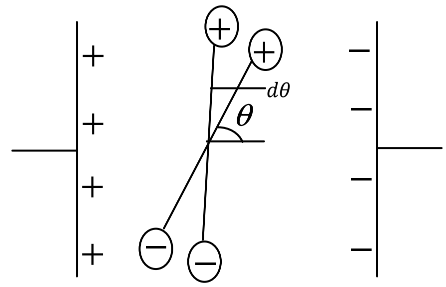
when θ = 90º Max. value of torque p E.



$\hat{\tau}$, Torque is into the plane of the paper

Q20. Derive an expression for potential energy of an electric dipole in a uniform electric field.





Ans. Work & Potential Energy

Work = $\int \tau \, d\theta$

$= \int PE \sin\theta \, d\theta$

$= -[PE \cos\theta]_0^\theta$

W = PE (1- Cos θ)

= PE (1-cos θ) – PE (1-cos 90)

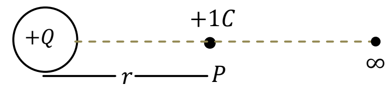
= - P.E. cos θ

U = $-\vec{P}.\vec{E}$

Q21. a) What is Electric Potential? Units? Dimensions?
b) What is Electric Potential difference? Example?

Ans.a) **Electric Potential**:-

A unit +ve test charge is place at point *P* at distance r from +Q charge. Charge *Q* is large and at rest. Charge at *P* will move from *P* to ∞ due to force of repulsion.



"Work done by the unit test charge as it moves from *P* to ∞, is called potential at point *P*".

OR

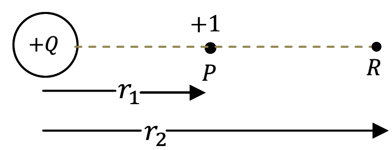
"Work to be done on the charge by external agent to move unit test charge from ∞ to point *P*".

Example:- If 100 Joule of work is done by unit test charge, potential is $100 {}^{J}/_{C}$ i.e. 100 volt.

Units:- ${}^{J}/_{Coulomb}$ i.e. volt.

Dimensions:- $[V] = \frac{Work}{Charge} = \frac{ML^2T^{-2}}{A.T} = M^1.L^2T^{-3}.A^{-1}$

b) **Potential Difference**:-



Potential difference between two point *P* and *R* is defined as amount of work which a unit +ve test charge will do as it moves from point P to *R*.

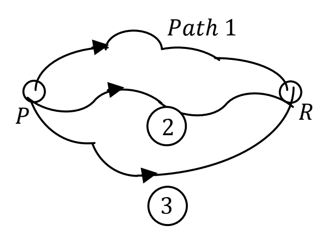
i.e. $\Delta V = V_P - V_R$

V_P is potential at *P* V_R is potential at *R* ΔV is potential difference

Example:- $\Delta V = V_P - V_R$

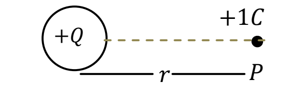
= (150 volt) – (100 volt)

ΔV = 50 volt



Potential difference between two points does not depend on path followed because electric field is conservative field.

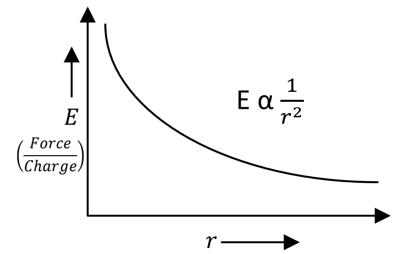
Q22. Plot variation of Electric Field and Electric Potential due to charge *+Q* v/s distance *r*.



Ans. **<u>Electric Field v/s *r*</u>**:-

As per Coulomb's Law

F $= \frac{1}{4\pi\epsilon_0} \cdot \frac{Q.1}{r^2}$



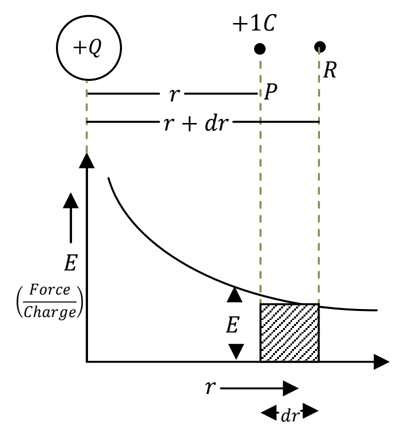
F on charge +1*C* is called *E*.

E	$= \frac{Q}{4\pi\epsilon_0 . r^2}$

E $\alpha \frac{1}{r^2}$

<u>Electric Potential</u>:-

Potential at a point P is amount of work done in taking a unit +ve test charge from that point to ∞, small work done in moving charge from *P* to *R*.



dw = F.dr

w $= \int_r^\infty F.dr$

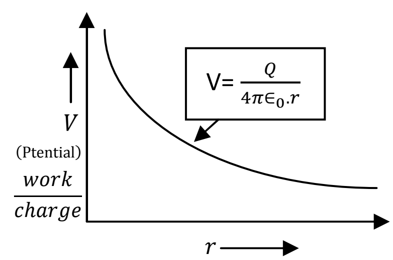
$= \int E.dr$ $\quad [|E| = |F|]$

$= \int_r^\infty \frac{Q}{4\pi\epsilon_0 . r^2} . dr$

$= \frac{Q}{4\pi\epsilon_0} . \int_r^\infty r^2 . dr$

$= \frac{Q}{4\pi\epsilon_0} \left| \frac{r^{-1}}{-1} \right|_{r = r}^{r = \infty}$

w	$= \frac{Q}{4\pi\epsilon_0 . r}$



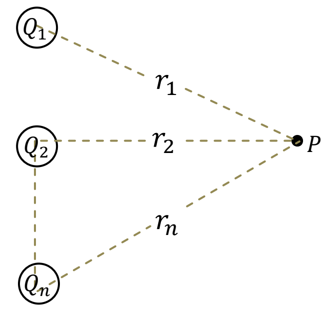
Work done w is for unit +ve test charge and is called electric potential.

V	$= \frac{Q}{4\pi\epsilon_0 . r}$

Q23. a) Derive an expression for potential at a point due to *n* charges.

b) Find potential at centre of a square having charges +1C, +2C, +3C, +4C at its corners. Each side of square is *L*.

Ans.a) **<u>Potential due to *n* charges</u>**:-



Potential due to charge Q, is

$$V = \frac{Q}{4\pi\epsilon_0 . r}$$

Potential due to Q_1, $V_1 = \frac{Q_1}{4\pi\epsilon_0 . r_1}$

Potential due to Q_2, $V_2 = \frac{Q_2}{4\pi\epsilon_0 . r_2}$

Potential due to Q_n, $V_n = \frac{Q_n}{4\pi\epsilon_0 . r_n}$

Total potential at P, V_P = Potential due to Q_1

+ Potential due to Q_2

+ Potential due to Q_n

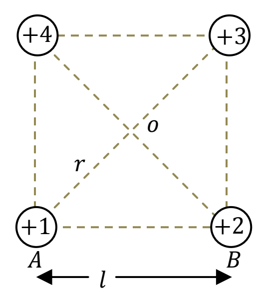
$$V_P = V_1 + V_2 \text{------------} V_n$$

$$= \frac{Q_1}{4\pi\epsilon_0 . r_1} + \frac{Q_2}{4\pi\epsilon_0 . r_2} \text{----------} \frac{Q_n}{4\pi\epsilon_0 . r_n}$$

$$V_P = \left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2} - - - - - - - - \frac{Q_n}{r_n}\right) \times \frac{1}{4\pi\epsilon_0}$$

Total potential is simple algebraic sum because potential is scalar quantity.

b) **<u>Total Potential at *O*, *V*</u>**:-



$$= V_A + V_B + V_C + V_D$$

$$= \frac{Q_A}{4\pi\epsilon_0 . r} + \frac{Q_B}{4\pi\epsilon_0 . r} + \frac{Q_C}{4\pi\epsilon_0 . r} + \frac{Q_D}{4\pi\epsilon_0 . r}$$

$$= \frac{1}{4\pi\epsilon_0 . r} . [Q_A + Q_B + Q_C + Q_D]$$

$$= \frac{1}{4\pi\epsilon_0 . r} . [1 + 2 + 3 + 4]$$

$$V = \frac{10}{4\pi\epsilon_0 . r}$$

$$V = \frac{10}{4\pi\epsilon_0 . \left(l/\sqrt{2}\right)}$$

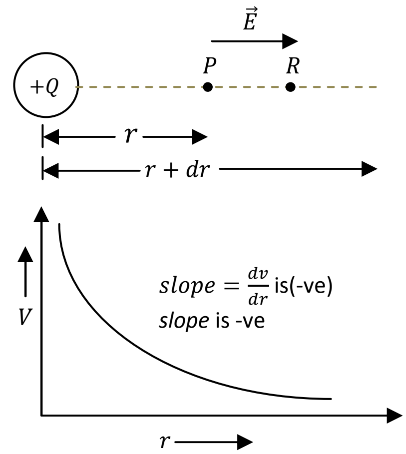
$$V = \frac{10\sqrt{2}}{4\pi\epsilon_0 . l}$$

Q24. What is relation between $\vec{E}$ and V.

a) For 1-Dimension

b) For 3-Dimension

Ans.a) **1-Dimension**:-



$$V_P = V + \Delta V$$

$$V_R = V$$

$$\Delta V = \frac{\Delta W}{+1C}$$

$$= \left(\frac{\vec{F}}{+1C}\right).\overrightarrow{dr}$$

$$\boxed{\Delta V = \vec{E}.\overrightarrow{dr}}$$

$$\boxed{\vec{E} = \frac{-dv}{dr}.\hat{r}}$$ because $\frac{dv}{dr}$ is –ve.

For *x-axis*, $$\boxed{\vec{E} = \frac{-dv}{dx}.\hat{\imath}}$$

b) ***V* is function of *x, y, z***:-

$$\vec{E} = \left(\frac{-dv}{dx}.\hat{\imath}\right) + \left(\frac{-dv}{dy}.\hat{\jmath}\right) + \left(\frac{-dv}{dz}.\hat{k}\right)$$

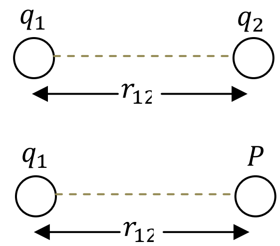
$$\vec{E} = \frac{-dv}{dx}.\hat{\imath} + \frac{-dv}{dy}.\hat{\jmath} + \frac{-dv}{dz}.\hat{k}$$

$$\vec{E} = -\left[\hat{\imath}.\frac{d}{dx} + \hat{\jmath}.\frac{d}{dy} + \hat{k}.\frac{d}{dz}\right]V$$

$$\boxed{\vec{E} = -\vec{\nabla} V}$$

Q25. Write expression for electric potential energy due to
a) Two charges
b) Three charges
c) Four charges

Ans.a) **Two charges**:-



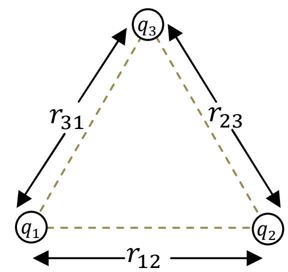
V at 2 due to 1 $= \frac{q_1}{4\pi\epsilon_0 . r_{12}}$

Electric potential energy, U_{12} = (V) q_2

$$U_{12} = \frac{q_1 . q_2}{4\pi\epsilon_0 . r_{12}}$$

b) **Three charges**:-

Total energy potential energy (U) = $U_{12} + U_{23} + U_{31}$



$$\therefore U = \frac{q_1 . q_2}{4\pi\epsilon_0 . r_{12}} + \frac{q_2 . q_{23}}{4\pi\epsilon_0 . r_{23}} + \frac{q_3 . q_1}{4\pi\epsilon_0 . r_{31}}$$

$$U = \frac{1}{4\pi\epsilon_0} . \left[\frac{q_1 . q_2}{r_{12}} + \frac{q_2 . q_{23}}{r_{23}} + \frac{q_3 . q_1}{r_{31}}\right]$$

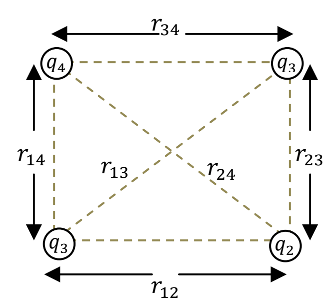
c) **Four charges**:-

Total energy potential energy (U) = ?

$$\therefore U = U_{12} + U_{23} + U_{13} + U_{14} + U_{24} + U_{34}$$

$$U = \frac{q_1 . q_2}{4\pi\epsilon_0 . r_{12}} + \frac{q_2 . q_3}{4\pi\epsilon_0 . r_{23}} + \frac{q_1 . q_2}{4\pi\epsilon_0 . r_{13}} + \frac{q_1 . q_4}{4\pi\epsilon_0 . r_{14}} + \frac{q_2 . q_4}{4\pi\epsilon_0 . r_{24}} + \frac{q_3 . q_4}{4\pi\epsilon_0 . r_{34}}$$

$$U = \frac{1}{4\pi\epsilon_0} . \left[\frac{q_1 . q_2}{r_{12}} + \frac{q_2 . q_3}{r_{23}} + \frac{q_1 . q_2}{r_{13}} + \frac{q_1 . q_4}{r_{14}} + \frac{q_2 . q_4}{r_{24}} + \frac{q_3 . q_4}{r_{34}}\right]$$



- Maximum number of diagonals = n_{c_2} - n
  As above = $4_{c_2} - 4 = 6 - 4 = 2$

Q26. **a) Derive an expression for potential due to Electric Dipole?**
b) Derive an expression for Electric Field due to Electric Dipole?

Ans.a) **Electric Potential**:-

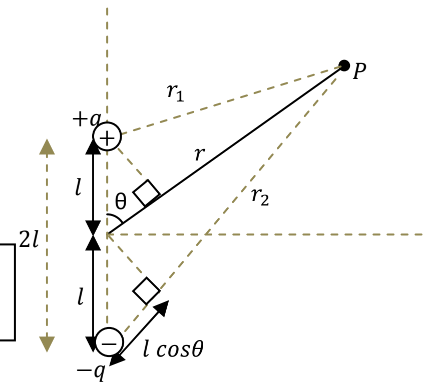
$$= \frac{+q}{4\pi\epsilon.r_1} + \frac{-q}{4\pi\epsilon_0.r_2}$$

$$= \frac{q}{4\pi\epsilon}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= \frac{q}{4\pi\epsilon}\left(\frac{1}{-l\cos\theta}\right) - \left(\frac{1}{r+l\cos\theta}\right)$$

$$= \frac{q}{4\pi\epsilon}\left(\frac{2l\cos\theta}{r^2-l^2\cos^2\theta}\right)$$

$r_1 = r - l\cos\theta$ $r_2 = r - l\cos\theta$



$$V_P = \frac{(q)(2l)\cos\theta}{4\pi\epsilon.r^2} \quad (r \gg l)$$

$$\boxed{V_P = \frac{P\cos\theta}{4\pi\epsilon.r^2}}$$ (where p=q.2 l)

b)

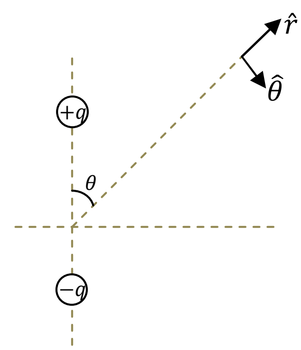
$$E = \frac{-dv}{dr}$$

$$\vec{E} = -\vec{\nabla} V$$

$$= -\left[\left(\frac{dv}{dr}\right)\hat{r} + \left(\frac{dv}{r.d\theta}\right)\hat{\theta}\right]$$

$$= -\left[\left(\frac{-2p\cos\theta}{4\pi\epsilon_0.r^3}\right)\hat{r} + \left(\frac{-p\sin\theta}{4\pi\epsilon_0.r^3}\right)\hat{\theta}\right]$$

$$\vec{E} = \left[\left(\frac{2p\cos\theta}{4\pi\epsilon_0.r^3}\right)\hat{r} + \left(\frac{p\sin\theta}{4\pi\epsilon_0.r^3}\right)\hat{\theta}\right]$$



$$V = \frac{p\cos\theta}{4\pi\epsilon.r^2}$$

$$\boxed{V = \frac{p\cos\theta}{4\pi\epsilon.r^2}} \qquad \boxed{\vec{E} = \left(\frac{2p\cos\theta}{4\pi\epsilon_0.r^3}\right)\hat{r} + \left(\frac{p\sin\theta}{4\pi\epsilon_0.r^3}\right)\hat{\theta}}$$

Case I:- Find *V* and *E* on *equatorial line*, θ=90

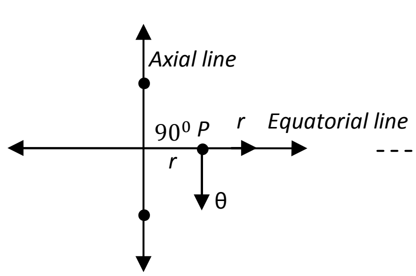
$$V_P = \frac{p\cos 90^0}{4\pi\epsilon.r^2} = 0$$

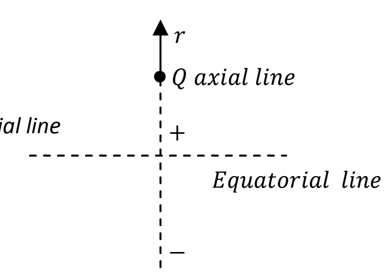
$$\vec{E}_P = \frac{p}{4\pi\epsilon.r^3} . \hat{\theta}$$

Case II:-Find *V* and *E* on *axial line*, θ=0

$$V_Q = \frac{p}{4\pi\epsilon.r^2}$$

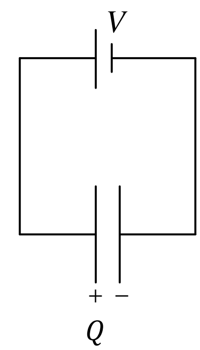
$$\vec{E}_Q = \frac{2p}{4\pi\epsilon.r^3} . \hat{r}$$





Q27. a) What is capacitance? SI Units? Dimensions?
b) What is capacitance of a sphere of radius *R*?

Ans.a) **Capacitance**:-



Voltage *V* across a capacitor can create charge *Q* on plates.

If voltage across plates increase charge *Q* also increases.

i.e. $Q \propto V$

$Q = C.V$ C→ constant of proportionality

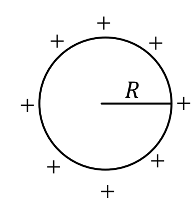
$$C = Q/V$$

***C* is called capacitance.**

SI Unit:- 1 farad $\left[i.e.\ 1\,\frac{coul}{volt}\right]$

Dimensions:- $C = \frac{[Q]}{[V]} = \frac{A.T}{\frac{ML^2T^{-2}}{A.T}} = [M^{-1}.L^{-2}.T^4.A^2]$

b) **Capacitance of sphere**:-



Charge *Q* is placed on a metal

Sphere of radius *R*

Voltage on surface, $V = \frac{Q}{4\pi\epsilon_0.R}$

$\frac{Q}{V} = 4\pi.\epsilon_0.R$

$$C = 4\pi.\epsilon_0.R$$

Example:-

Capacitance of Earth

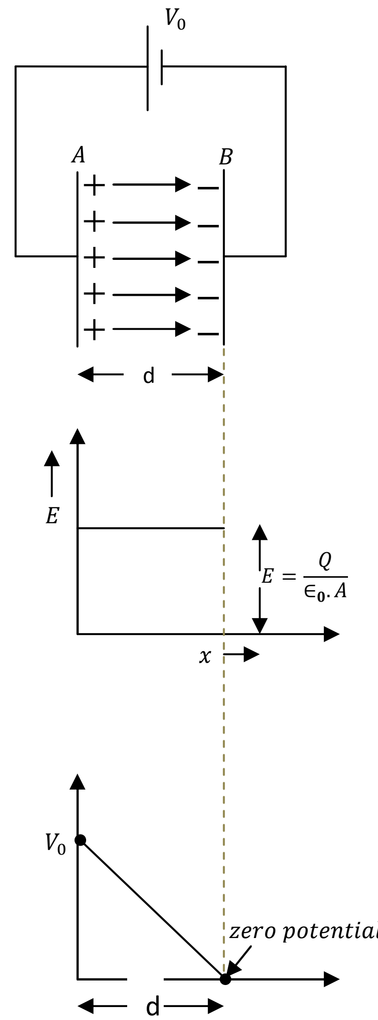
$C = 4\pi.\epsilon_0.R$

$= 4\pi.\epsilon_0$ (6400 Km)

$$C_{earth} = 714\mu F$$

Q28. Prove capacitance of parallel plate. Capacitor is $\frac{\epsilon_0 . A}{d}$.

Ans. Apply voltage V_0 across two metal plates A and B. +ve charge Q appears on A and –ve charge $-Q$ on B.



We are to find $\boxed{C = \frac{Q}{V}}$ ----------------- ①

Electric lines start from A and end up at B.

Number of lines/area remain same between two plates.

So, $E = \frac{No. of\ lines}{Area}$ (Constant here)

$$E = \frac{Q/\epsilon_0}{A}$$

$$\boxed{E = \frac{Q}{\epsilon_0 . A}}$$

$$\frac{-dv}{dx} = \frac{Q}{\epsilon_0 . A} \quad \left[\text{As } E = \frac{-dv}{dx}\right]$$

$$\frac{\Delta v}{d} = \frac{Q}{\epsilon_0 . A}$$

$$\Delta v = Q . \frac{d}{\epsilon_0 . A}$$

$$\boxed{\frac{Q}{\Delta v} = \frac{\epsilon_0 . A}{d}} \text{ ----------------- ②}$$

From ① and ②

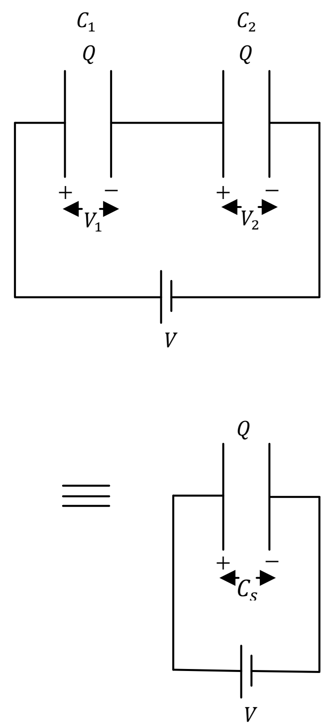
$\boxed{\text{Capacitance, } C = \frac{\epsilon_0 . A}{d}}$ Also

$C = \epsilon_0 . \epsilon_r . \frac{A}{d}$ $\epsilon_r \rightarrow$ Relative permittivity of material between plates

Q29. Prove:

a) For 2 capacitors in series, $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2}$

b) For 2 capacitors in parallel, $C_P = C_1 + C_2$.



Ans.a) As two capacitors are in series, current flows for same time at time of switching.

$$Q = I \, . \, t$$

I and t are same for two capacitors in series.

So, $\boxed{Q \text{ is same}}$

$$V = V_1 + V_2 \quad \text{(As per Kirchoff's Voltage Law)}$$

$$\Rightarrow \frac{Q}{C_S} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\Rightarrow \boxed{\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2}}$$ for two capacitors in series.

b) Total charges supplied by the battery Q is divided on two paths.

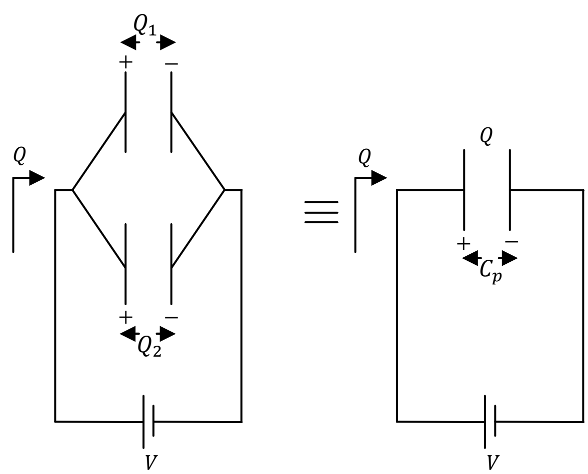
Q_1 flows to C_1 and Q_2 to C_2

$$Q = Q_1 + Q_2$$

$$C_p.V = C_1.V + C_2.V$$

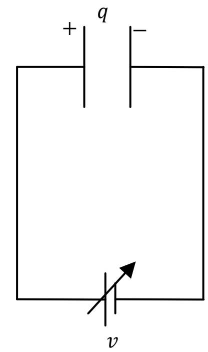
$$\boxed{C_p = C_1 + C_2}$$

for two capacitors in parallel.



Q30. Prove energy stored in a capacitor is $\frac{1}{2}.C.V^2$.

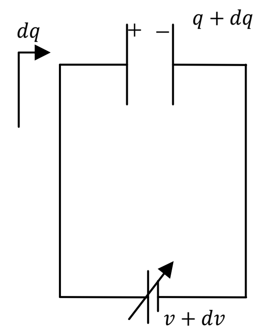
Ans. A variable voltage source is connected across a capacitor as shown in *fig*.



A voltage causes a charge q on plates.

Voltage is increased by dv. Extra charge dq flows to capacitor.

Battery does some work in pushing this charge dq on plates.



$$dw = v.dq \qquad \left[v \rightarrow \frac{work}{charge}\right]$$

$$w = \int v.dq$$

$$= \int \frac{q}{C}.dq \qquad \left(As\ c = \frac{q}{v}\right)$$

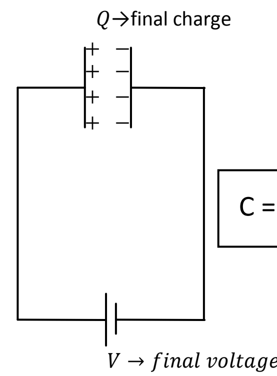
$$= \left|\frac{q^2}{2.C}\right|_{q=0}^{q=Q}$$

$$\boxed{w = \frac{Q^2}{2.C}} \quad \text{------------ (1)}$$

OR

$$w = \frac{(C.V)^2}{2.c} = \frac{1}{2}.C.V^2$$

$$\boxed{w = \frac{1}{2}.C.V^2} \quad \text{------------ (2)}$$



C = $\frac{Q}{V}$

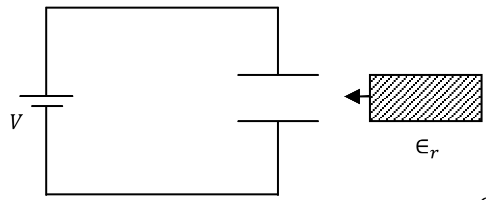
$$w = \frac{1}{2}.\frac{Q}{V}.V^2 = \frac{1}{2}.Q.V$$

i.e. $$\boxed{C = \frac{1}{2}.Q.V} \quad \text{------------ (3)}$$

Q31. A dielectric slab is introduced between plates of a capacitance with battery. What is effect on Capacitance *C*, Voltage *V*, Charge *Q*, Energy stored *U*.

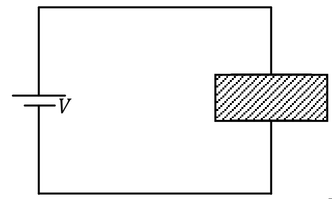
Ans.

C without dielectric



Charge, Q
Voltage, V
Capacitance, C
Energy, U

C with dielectric



Charge, Q'
Voltage, V'
Capacitance, C'
Energy, U'

a) **Capacitance, *C***:-

As $C = \in_0 . \in_r . \frac{A}{d} = \in_0 . 1 . \frac{A}{d}$

$C' = \in_0 . \in_r . \frac{A}{d}$

$= \in_0 . \left(\in_r . {}^{A}/_{d}\right)$

$$\boxed{C' = \in_r . C}$$

Capacitance becomes $\in_r$ times after inserting the slab.

b) **Voltage, *V***:-

As battery is connected in both cases, so voltage, *V* remains same.

$\boxed{V' = V}$ No change in voltage across capacitor.

c) **Charge, *Q***:-

$C = \frac{Q}{V} \Rightarrow \boxed{Q = C . V}$

$Q' = C' . V'$

$= (\in_r . C)(V)$

$= \in_r .(C.V)$

$\boxed{Q' = \in_r . Q}$ Charge becomes $\in_r$ times

d) **Energy, *U***:-

Energy, $U = \frac{1}{2} . C . V^2$

$U' = \frac{1}{2} . C' . (V')^2$

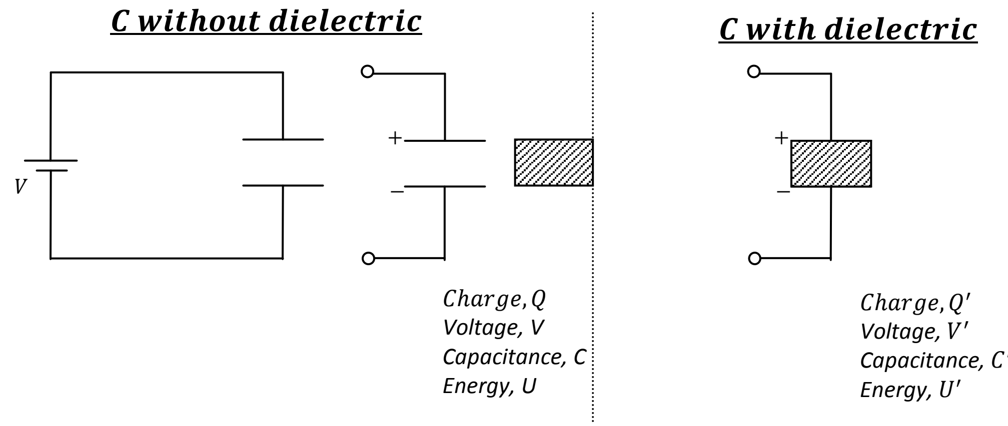
$= \frac{1}{2} . (\in_r . C) . (V)^2$

$U' = \in_r . \left(\frac{1}{2} . C . V^2\right)$

$\Rightarrow \boxed{U' = \in_r . U}$ Energy becomes $\in_r$ times

Q32. A dielectric slab is introduced between plates of a capacitor with battery disconnected. What is effect on Capacitance *C*, Voltage *V*, Charge *Q*, Energy stored *U*.

Ans.



a) **Capacitance, *C*:-**

$$C = \epsilon_0 . \frac{A}{d}$$

$$C' = \epsilon_0 . \epsilon_r . A/d \Rightarrow \boxed{C' = \epsilon_r . C}$$

Capacitance becomes ϵ_r times.

b) **Voltage, *V*:-**

$$C' = \frac{Q'}{V'} \quad \text{and} \quad C = \frac{Q}{V}$$

$$V' = \frac{Q'}{C'} \quad \text{and} \quad V = \frac{Q}{C}$$

So, $$\frac{V'}{V} = \frac{Q'/C'}{Q/C} = \left(\frac{Q'}{Q}\right)\left(\frac{C}{C'}\right) = (1)\left(\frac{C}{\epsilon_r . C}\right) = \frac{1}{\epsilon_r}$$

$$\frac{V'}{V} = \frac{1}{\epsilon_r} \Rightarrow \boxed{V' = \frac{V}{\epsilon_r}}$$

Voltage becomes $\frac{1}{\epsilon_r}$ times.

c) **Charge, *Q*:-**

As circuit is open, charge cannot flow in the loop.

$$\boxed{Q' = Q}$$

Charge on plates remains same.

d) **Energy, *U*:-**

Energy, $U = \frac{1}{2} . C . V^2$

$$U' = \frac{1}{2} . C' . (V')^2$$

$$\frac{U'}{U} = \frac{\frac{1}{2} . C' . (V')^2}{\frac{1}{2} . C . V^2} = \left(\frac{C'}{C}\right)\left(\frac{V'}{V}\right)^2 = \left(\frac{\epsilon_r . C}{C}\right)\left(\frac{1}{\epsilon_r}\right)^2 = \frac{1}{\epsilon_r}$$

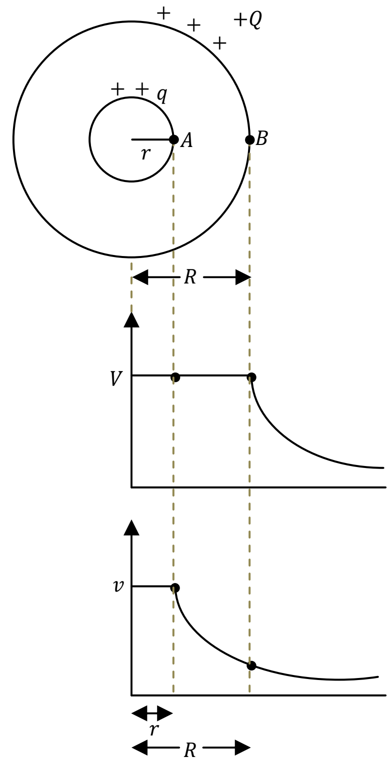
$$\boxed{\frac{U'}{U} = \frac{1}{\epsilon_r}}$$

Energy becomes $\frac{1}{\epsilon_r}$ times.

Q33. **Explain Principle, construction and working of *VAN DE GRAAFF GENERATOR*.**

Ans.a) **<u>Principle</u>**:-

Charge *Q* is placed on outer sphere of radius *R*. Charge *q* is place on inner sphere of radius *r*.



Potential at *A*, $V_A = \frac{Q}{4\pi\epsilon_0 . R} + \frac{q}{4\pi\epsilon_0 . r}$

Potential at *B*, $V_B = \frac{Q}{4\pi\epsilon_0 . R} + \frac{q}{4\pi\epsilon_0 . R}$

Potential difference, $$V_A - V_B = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r} - \frac{1}{R}\right)$$

Potential difference does not depend on Q.

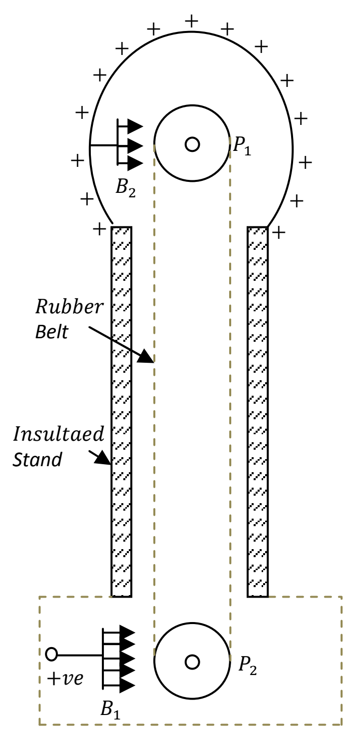
If *q* is +ve, $V_A > V_B$. If now *A* and *B* are connected, charge will flow from *A* to *B* for all values of *q* or *Q*.

b) **<u>Construction</u>**:-

P_1, P_2 → Pulleys (to rotate the belt)

B_1 →Brush used to spay charge on belt

B_2 →Brush used to collect charge from belt



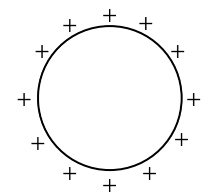
c) **<u>Working</u>**:-

Brush B_1 sprays +ve charge on the rotating belt. This charge is carried to brush B_2, transfers this charge to sphere. Charge on sphere keeps on increasing till it creates electric field of the order of 30 $^{kv}/_{cm}$ near surface of sphere. For sphere size of the order of few meters, voltage of order of 60Lac volt is created using above arrangement.

<u>Use</u>:- Such high voltage are further used to accelerate charged particles.

Q34. Write electrostatic properties of conductors?

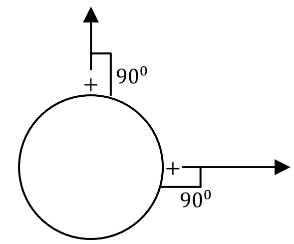
Ans.1. **Inside a conductor, electrostatic field is ZERO**:-



All charges reside on the surface of a conductor. If E is not zero inside the conductor, E will exert a force on charge and move it to surface.

2. **$\vec{E}$ is normal to surface**:-

Let us assume $\vec{E}$ is not normal to surface.



$E_T = E.\cos\theta$ is tangential component so, E_T will exert a force on charge and will move q along surface.

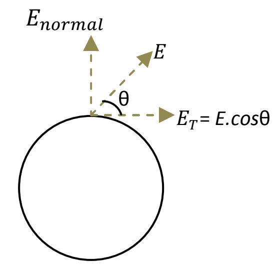
As charge is at rest,

So, $E_T = 0$

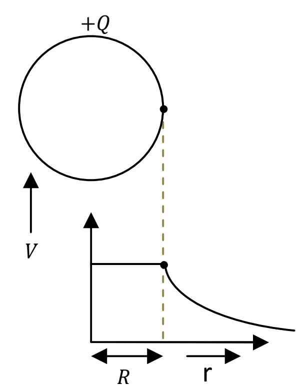
$\Rightarrow E.\cos\theta = 0$

$\Rightarrow \cos\theta = 0$

$\Rightarrow \boxed{\theta = \pi/2}$



3. **Electrostatic potential is constant inside a conductor**:-



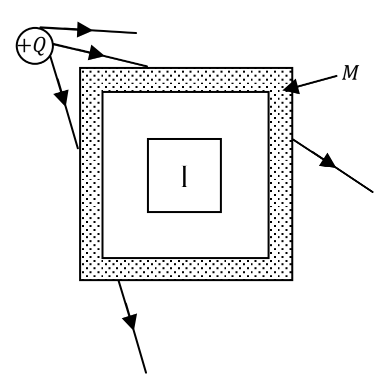
Electrostatic potential is constant throughout the object and its value is same as that on the surface.

$E_{in} = 0$

and $V_{in} = V_{surface} = \frac{Q}{4\pi\epsilon_0.R}$

4. **Electrostatic shielding**:-

Electric lines from charge *Q* disturb electrical/electronic instrument *I*.



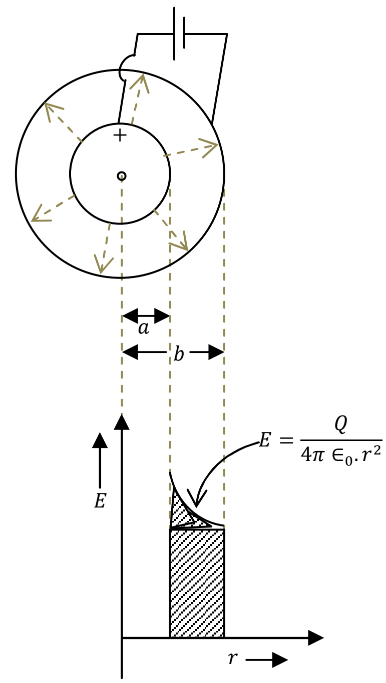
Electrical/electronic instrument *I* is surrounded by metallic enclosure *M*.

Electric lines get bye-passed from metallic enclosure *M* without disturbing the instrument *I*.

This effect of placing metallic enclosure surrounding an instrument is called *SHIELDING*.

Q35. Find capacitance of concentric spheres of radius '*a*' and '*b*'.

Ans. $C = \frac{charge}{voltage}$



Shaded Area gives potential difference between two spheres

Step 1:-

Connect voltage *V* across two concentric spheres with inner radius '*a*' and outer radius '*b*' as shown in *fig*.

Step 2:-

Find $\vec{E}$ for a ≤ r ≤ b

$E = \frac{no.of\ lines}{area}$

$= \frac{Q/\in_0}{4\pi r^2}$

$E = \frac{Q}{4\pi \in_0 . r^2}$

Step 3:-

Pot. Difference between two spheres, ΔV

$\Delta V = \int E.dr$ [Area under *E*-*r* graph]

$= \int \frac{Q}{4\pi \in_0 . r^2} . dr$

$= \frac{Q}{4\pi \in_0} \int r^{-2} . dr$

$= \frac{Q}{4\pi \in_0} \left| \frac{-1}{r} \right|_{r=a}^{r=b}$

$= \frac{Q}{4\pi \in_0} \left(\frac{1}{a} - \frac{1}{b} \right)$

$$\boxed{\Delta V = \frac{Q}{4\pi \in_0} \left(\frac{b-a}{ab} \right)}$$

Step 4:-

$\frac{Q}{\Delta V} = 4\pi \in_0 . \in_r \left(\frac{ab}{b-a} \right)$

$$\boxed{C = \frac{Q}{\Delta V} = 4\pi \in_0 . \in_r \left(\frac{ab}{b-a} \right)}$$

$$\boxed{C_{sphere} = 4\pi \in_0 . \in_r \left(\frac{Product\ of\ radii}{Diff.of\ radii} \right)}$$

Q36. Find capacitance of co-axial cylinders.

Ans. **Step 1**:-

Apply voltage across the two cylinders, each of length 'L' under consideration.

Step 2:-

Find Electric Field

Find $\vec{E}$ for a ≤ r ≤ b

$$E = \frac{Q/\epsilon_0}{2\pi r.L}$$

$$= \frac{Q/L}{2\pi\epsilon_0.r}$$

$$\boxed{E = \frac{\lambda}{2\pi\epsilon_0.r}}$$

Step 3:-

Find Potential difference, ΔV

$$\Delta V = \int E.dr$$

Area under E-r graph Gives potential difference

$$= \int \frac{\lambda}{2\pi\epsilon_0.r}.dr$$

$$= \frac{\lambda}{2\pi\epsilon_0}\int \frac{1}{r}.dr$$

$$= \frac{\lambda}{2\pi\epsilon_0}\left|log_e.r\right|_{r=a}^{r=b}$$

$$\boxed{\Delta V = \frac{\lambda}{2\pi\epsilon_0}.log_e\left(b/a\right)}$$

Step 4:-

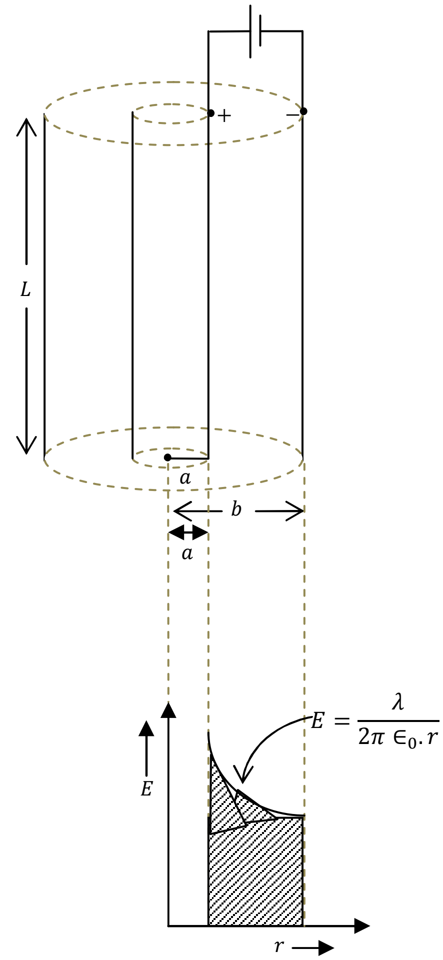
$$\Delta V = \frac{Q/L}{2\pi\epsilon_0}.log_e\left(b/a\right)$$

$$\frac{Q}{\Delta V} = \frac{2\pi\epsilon_0.L}{log_e\left(b/a\right)}$$

$$\boxed{C = \frac{2\pi\epsilon_0.L}{log_e\left(b/a\right)}}$$

$$\boxed{\frac{C}{L} = \frac{2\pi\epsilon_0}{log_e\left(b/a\right)}}$$

$\frac{C}{L}$ → Capacitance per unit length



Shaded Area gives potential difference between two cylinders